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# REALISM, RATIOCINATION, AND RULES

KEVIN W. SAUNDERS\*

## I. Introduction

The literature in jurisprudence shows a recent interest in the question of what mathematics, or more properly metamathematics,<sup>1</sup> may say about the nature of law.<sup>2</sup> Particularly, scholars have attempted to apply theories regarding the indeterminacy of mathematics<sup>3</sup> to law in order to show that law must also be indeterminate.

Professors Rogers and Molzon,<sup>4</sup> Brown and Greenberg,<sup>5</sup> and D'Amato,<sup>6</sup> among others, have all sought to draw some insight into law through an examination of Godel's Theorem.<sup>7</sup> Godel's Theorem demonstrates that any formalization of arithmetic will be incomplete. That is, no matter what axioms one chooses as the basis from which to prove the truths of arithmetic, there will always exist propositions that can neither be proved true nor false. There will always be gaps, and the addition of further axioms for arithmetic will not fill the gaps. Thus, an infinity of unprovable propositions will always remain.

Professor D'Amato concludes that Godel's Theorem carries over to law and demonstrates that there will always be an infinity of legal propositions that can be neither proved nor disproved.<sup>8</sup> Brown and Greenberg similarly conclude that "Godel's Theorem reveals that the law cannot be a determinate formal system."<sup>9</sup>

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1. Metamathematics is the study of mathematics rather than study within mathematics. It is the examination of the nature of mathematics rather than the derivation of propositions within particular mathematical systems.

2. See, e.g., John M. Farago, *Intractable Cases: The Role of Uncertainty in the Concept of Law*, 55 N.Y.U. L. REV. 195 (1980); John M. Farago, *Judicial Cybernetics: The Effects of Self-Reference in Dworkin's Rights Thesis*, 14 VAL. U. L. REV. 371 (1980). See *infra* notes 4-6.

3. See *infra* notes 7-25 and accompanying text.

4. John M. Rogers & Robert E. Molzon, *Some Lessons About the Law from Self-Referential Problems in Mathematics*, 90 MICH. L. REV. 992 (1992).

5. Mark R. Brown & Andrew C. Greenberg, *On Formally Undecidable Propositions of Law: Legal Indeterminacy and the Implications of Metamathematics*, 43 HASTINGS L.J. 1439 (1992).

6. Anthony D'Amato, *Can Legislatures Constrain Judicial Interpretation of Statutes?*, 75 VA. L. REV. 561 (1989) [hereinafter D'Amato, *Judicial Interpretation*].

7. For a discussion of Godel's Theorem and an explanation of the proof, see generally ERNEST NAGEL & JAMES R. NEWMAN, *GODEL'S PROOF* (1958).

8. D'Amato, *Judicial Interpretation*, *supra* note 6, at 597.

9. Brown & Greenberg, *supra* note 5, at 1487. While Brown and Greenberg insert the word "formal" and might then be read to leave open the possibility that law is a determinate informal system, D'Amato argues that if law is a nonformal system, it must for that reason be indeterminate. Anthony D'Amato, *Pragmatic Indeterminacy*, 85 NW. U. L. REV. 148, 175-76 n.92 (1990) [hereinafter D'Amato, *Pragmatic Indeterminacy*].

Professors Rogers and Molzon seem more reluctant to apply Godel's Theorem directly to law but state: "Godel's Theorem strongly suggests that it is impossible to create a legal system that is complete in the sense that there is a derivable rule for every fact situation."<sup>10</sup>

Professor D'Amato has also found import in mathematics' Lowenheim-Skolem Theorem that any axiom set for an area of mathematics will admit an infinite variety of alternate interpretations.<sup>11</sup> That is, in any attempt to develop a set of axioms from which the truths of a particular area of mathematics may be derived, there will be alternative mathematical systems that satisfy the same axioms. D'Amato combines the Lowenheim-Skolem Theorem with other mathematical or philosophical arguments to conclude that "even a highly formalized set of rules, such as the Restatements of Contracts and Torts, can consistently be said to 'apply' to mutually inconsistent descriptions of fact situations."<sup>12</sup>

Professor Ken Kress attacks these efforts to use the theorems of metamathematics to draw conclusions about the nature of law.<sup>13</sup> He points out that the Lowenheim-Skolem Theorem applies to rigorously defined formal systems and that

English[] and legal language are insufficiently precise for the assertions and inferences of a formal proof such as the Lowenheim-Skolem Theorem to be true and valid about them. As mathematicians put it, the Lowenheim-Skolem proof will not go through in legal English. There is therefore no reason to suppose that the conclusion of the proof, the Lowenheim-Skolem Theorem, is true in legal English . . . .<sup>14</sup>

In response, D'Amato continues to assert the relevance of the Lowenheim-Skolem proof, but he also argues that if legal English is so informal as to not allow the proof, that also demonstrates indeterminacy.

While Professor Kress specifically attacks the use of the Lowenheim-Skolem Theorem, similar objections apply to the application of Godel's Theorem to law. Godel's Theorem works in mathematics, because Godel managed to express the metalanguage for arithmetic, the language used to talk about arithmetic, within arithmetic.<sup>15</sup> In order for Godel's Theorem to apply to law, it would seem that the same feat must be accomplished for law. More is required than showing that laws may be self-referential or that law may have rules and metarules.<sup>16</sup> Required is a demonstration that the metalanguage of law — legal English — can in some sense be embedded in the law. It is certainly not obvious that that can be accomplished; indeed, it may even be unclear what it means.

10. Rogers & Molzon, *supra* note 4, at 992.

11. Anthony D'Amato, *Can Any Legal Theory Constrain Any Judicial Decision?*, 43 U. MIAMI L. REV. 513, 521 n.28 (1989).

12. D'Amato, *Pragmatic Indeterminacy*, *supra* note 9, at 175-76.

13. Ken Kress, *A Preface to Epistemological Indeterminacy*, 85 NW. U. L. REV. 134 (1990).

14. *Id.* at 144.

15. See NAGEL & NEWMAN, *supra* note 7.

16. See Rogers & Molzon, *supra* note 4, at 1002-21.

Professors Rogers and Molzon do, however, draw an important nonanalytic conclusion from their examination of Godel's Theorem. Even if Godel's Theorem does prove the incompleteness of law, they note that "legal theorists must become comfortable with the incompleteness of legal systems, no matter how carefully constructed, in the same way that mathematicians and philosophers have become comfortable with the incompleteness of axiomatic systems of number theory."<sup>17</sup> That does seem to be an important observation. While it is questionable what formal logic results of mathematics carry over to law, the psychological observation that incompleteness has not hobbled practitioners of mathematics, the seeming paradigm of certainty, may carry over to relieve our anxieties over incompleteness in the area of law.<sup>18</sup>

That last observation of Rogers and Molzon also raises the issue that is the focus of this article — an exploration of what the self-examination of mathematicians with regard to their work can do to address the indeterminacy concerns of the American legal realists. The American legal realist judges, in their exercises of self-examination, noted that they did not apply rules but instead intuited conclusions.<sup>19</sup> Only once the conclusion was in hand did the judges attempt to construct what at least would appear to be legal analysis leading to that conclusion.<sup>20</sup>

While the realists did not jump from their observation to the conclusion that there are no legal rules, others have not been so reticent.<sup>21</sup> If what would appear to be legal rules and principles actually play no role in reaching a legal conclusion or the outcome of a case, it might be argued they are not real rules. Rather than directing results, legal rules and principles would serve only as post hoc rationalizations. While it might be argued that they at least serve as limitations in that, while the principles are adopted post hoc, they must be found in preexisting law, it may be countered that there exists such a variety of competing and inconsistent principles that there is no conclusion that cannot find such post hoc justification.<sup>22</sup>

This article will first examine the claims of the legal realists and the later claims that there are no legal rules.<sup>23</sup> A parallel will then be drawn to the self-examination of mathematicians, who also note that they work backwards from conclusions to arguments for those conclusions.<sup>24</sup> Lastly, the comparison between legal and

17. *Id.* at 992.

18. It is important to note that incompleteness is a far lesser weakness than radical indeterminacy. Incompleteness asserts only that there are propositions that can not be proven either true or false. While there are an infinity of such propositions in any formalization of arithmetic, there are also propositions that are provable. Even if Godel's Theorem were to carry over to law, it would prove only that there are hard cases not formally decidable under precedent, not that all cases are undecidable. *See infra* notes 66-68 and accompanying text.

19. *See infra* notes 26-27, 32 and accompanying text.

20. *See infra* notes 28-31, 33-34 and accompanying text.

21. *See infra* notes 35-44 and accompanying text.

22. *See infra* notes 77-78 and accompanying text.

23. *See infra* notes 26-44 and accompanying text.

24. *See infra* notes 45-58 and accompanying text.

mathematical analysis will call into question the conclusion that there are no legal rules.<sup>25</sup>

## *II. Realism and Legal Analysis*

Judge Hutcheson, in *The Judgment Intuitive: The Function of the "Hunch" in Judicial Decision*,<sup>26</sup> an article in which the thesis is clear from the title, explained his method of reaching a decision in a legal case. He said, "I, after canvassing all the available material at my command, and duly cogitating upon it, give my imagination play, and brooding over the cause, wait for the feeling, the hunch — that intuitive flash of understanding which makes the jump-spark connection between question and decision . . . ."<sup>27</sup> The judge also noted that he was "speak[ing] of the judgment pronounced, as opposed to the rationalization by the judge on that pronouncement."<sup>28</sup>

With regard to the rationalization, the judge tells us that, while the

motivating impulse for the decision is an intuitive sense of what is right or wrong for that cause, . . . the astute judge, having so decided, enlists his every faculty and belabors his laggard mind, not only to justify that intuition to himself, but to make it pass muster with his critics.<sup>29</sup>

Further,

having traveled and reached his judgment, he struggles to bring up and pass in review before his eager mind all of the categories and concepts which he may find useful directly or by analogy, so as to select from them that which in his opinion will support his desired result. . . . [T]he judge . . . must at least appear reasonable, and unless he can find a category which will at least "semblably" support his view, he will feel uncomfortable.<sup>30</sup>

With regard to the direction of any analysis, he quotes Max Radin in describing the judge as "'working his judgment backward' as he blazes his trail 'from a desirable conclusion back to one or another of a stock of legal premises.'"<sup>31</sup>

Judge Jerome Frank reached much the same conclusion in his examination of the work of a judge. He commented on judging generally, writing:

The process of judging, so the psychologists tell us, seldom begins with a premise from which a conclusion is subsequently worked out.

25. See *infra* notes 59-88 and accompanying text.

26. Joseph C. Hutcheson, *The Judgment Intuitive: The Function of the "Hunch" in Judicial Decision*, 14 CORNELL L. Q. 274 (1929).

27. *Id.* at 278.

28. *Id.* at 279.

29. *Id.* at 285.

30. *Id.* at 286-87.

31. *Id.* at 287 (quoting Max Radin, *The Theory of Judicial Decision: Or How Judges Think*, 11 A.B.A. J. 357, 359 (1925)).

Judgment begins rather the other way around — with a conclusion more or less vaguely formed; a man ordinarily starts with such a conclusion and afterwards tries to find premises which will substantiate it.<sup>32</sup>

He argues that the situation is the same in law. "Judicial judgments, like other judgments, doubtless, in most cases, are worked out backward from conclusions tentatively formulated."<sup>33</sup>

Legal opinions, Judge Frank tells us, are misleading. "They picture the judge applying rules and principles to the facts, that is, taking some rule or principle (usually derived from opinions in earlier cases) as his major premise, employing the facts of the case as the minor premise, and then coming to his judgment by process of pure reasoning."<sup>34</sup> For Judge Frank, also, it is the conclusion that is adopted first, by intuition or hunch, and then the opinion is constructed by arguing backward from that conclusion to generally accepted premises.

The observations of the legal realists did not lead them all to reject the existence of legal rules. In fact, Judge Frank said: "It is sometimes asserted that to deny that law consists of rules is to deny the existence of legal rules. That is specious reasoning. . . . [W]hile rules are not the only factor in the making of law, i.e. decisions, that is not to say there are no rules."<sup>35</sup> Professor Llewellyn also argued that rules of law were obviously not eliminated but that the arguments of the realists explained the importance of the rules as "authoritative ought[s] addressed to officials" but to which the officials may "either pay no heed at all . . . or listen partly . . . or listen with all care . . . ."<sup>36</sup>

While the Realists themselves may have been reluctant to dismiss the existence of any real legal rules, some scholars of the critical legal studies movement have been less reticent. The "crits," like the realists, claim that "language is vague and . . . when two conflicting rules often cover the same factual setting, there is no separate mechanical rule as to which one governs."<sup>37</sup> In the crit view, "mainstream liberal thought . . . is simultaneously beset by internal *contradiction* (not by 'competing concerns' artfully balanced until a wise equilibrium is reached, but by irreducible, irremedial, irresolvable conflict) and by systematic *repression* of the presence of these contradictions."<sup>38</sup> As Professor Singer says, "Legal doctrines are always potentially indeterminant. Judges can move the line between rules and exceptions, or create new exceptions. . . . [T]he legal theories advanced to justify

32. JEROME FRANK, *LAW AND THE MODERN MIND* 108 (1930).

33. *Id.* at 109.

34. *Id.* at 111.

35. FRANK, *supra* note 32, at 141-42. Judge Frank has been characterized as finding such legal rules only marginally important in the conduct of trials, Charles M. Yablon, *Justifying the Judge's Hunch: An Essay on Discretion*, 41 HASTINGS L.J. 231, 237 (1990), but that does not deny their existence or importance in the appellate context, nor their importance in some jury trials.

36. Karl N. Llewellyn, *A Realistic Jurisprudence — The Next Step*, 30 COLUM. L. REV. 431, 449 (1930).

37. MARK KELMAN, *A GUIDE TO CRITICAL LEGAL STUDIES* 45 (1987).

38. *Id.* at 3.

our rules and institutions are indeterminant. The same theories could be used to justify very different sorts of institutions and very different rules.<sup>39</sup>

Sounding the same note, Professor Kelman attacks legal argument:

Most of the arguments that law professors make are not only nonsensical according to some obscure and unreachable criteria of Universal Validity but they are also patently unstable babble. The shakiness of the argumentative structure is, quite remarkably, readily elucidated. All the fundamental, rhetorically necessary distinctions collapse at a feather's touch . . . .<sup>40</sup>

And similarly, from Professor Peller:

The study of the underlying metaphysical assumptions of legal thought suggests that the purported distinction between rational legal argumentation and irrational emotional appeal is incoherent. Legal thought is merely one instance in a series of arational attempts to capture social experience in reproducible form. It is not qualitatively different from what it excludes as irrational.<sup>41</sup>

Professor Kelman would, however, distinguish the realists and the critics by noting the critics' stress is not only on the ambiguity of language but also on the psychological conflict that leads to the simultaneous commitment both to mechanical rules and to ad hoc standards.<sup>42</sup> Kelman appears to view the true heirs of the realists as those who, like Professors Fish<sup>43</sup> and D'Amato,<sup>44</sup> argue the nonexistence of rules based on the ambiguity of language. Whoever may be the true heirs of the realists, their influence is still felt, and the snowball of contradiction or indeterminacy that began with their self-reflective observations is still rolling.

## II. *The Mathematical Endeavor*

It is striking how similar the views of the American realist judges with regard to their work are to those of mathematicians with regard to their work. Both groups show concern that the nature of how they reach conclusions is not reflected in their published works. The judges assert that they reach the conclusion first and then work backwards to the premises that will justify that conclusion. Mathematicians

39. Joseph W. Singer, *The Player and the Cards: Nihilism and Legal Theory*, 94 YALE L.J. 1, 22-24 (1984).

40. Mark G. Kelman, *Trashing*, 36 STAN. L. REV. 293, 322 (1984).

41. Gary Peller, *The Metaphysics of American Law*, 73 CAL. L. REV. 1151, 1155 (1985).

42. KELMAN, *supra* note 37, at 3, 13, 44.

43. See, e.g., Stanley Fish, *Working on the Chain Gang: Interpretation in Law and Literature*, 60 TEX. L. REV. 551 (1982); Stanley Fish, *Wrong Again*, 62 TEX. L. REV. 299 (1983); Stanley Fish, *Still Wrong After All These Years*, 6 LAW & PHIL. 401 (1987).

44. See, e.g., Anthony D'Amato, *Pragmatic Indeterminacy*, *supra* note 9. Professor D'Amato acknowledges that the realists "gave us a taste of something profound" but notes that they lacked the insights of the writings of Wittgenstein, Goodman, Quine and Kripke. *Id.* at 154. The difference seems one of philosophical sophistication rather than the different focus Kelman claims.

also express concern that the theories they publish make it appear that they simply proceeded step by step from a set of axioms to the conclusion. They protest that, in fact, they reach their conclusions first and then see what axioms or theorems are required to prove that conclusion or whether or not the conclusion can be proved from a particular set of axioms.

Professor Polya compares the view of mathematics held by the non-mathematician and even some mathematics instructors with that of the research mathematician:

To some instructors, mathematics appears as a system of rigorous proofs . . . . To a mathematician, who is active in research, mathematics may appear sometimes as a guessing game: you have to guess a mathematical theorem before you prove it, you have to guess the idea of the proof before you carry through the details.<sup>45</sup>

Further, "[i]t may appear . . . surprising to the layman that the mathematician is . . . guessing. The result of the mathematician's creative work is demonstrative reasoning, a proof, but the proof is discovered by plausible reasoning, by guessing."<sup>46</sup> It is interesting to note that one of Professor Polya's most influential works in mathematics education was a film titled *Let Us Teach Guessing*.<sup>47</sup>

Professor Littlewood, one of Cambridge University's great mathematicians, describes the process of mathematical research and creation as having four phases: preparation, incubation, illumination, and verification.<sup>48</sup> While the preparation is a conscious activity, the incubation, Littlewood says, is a period of subconscious work. Of the other two phases, illumination and verification, Littlewood says that the verification, the working out of the proof, is "within the range of any competent practitioner, given the illumination."<sup>49</sup> On the other hand, "[i]llumination, . . . the emergence of the creative idea into the conscious[,] . . . implies some mysterious rapport between the subconscious and the conscious . . . ."<sup>50</sup>

It is the verification, the working out or proof, that appears in the mathematics journal or textbook, but for Littlewood that is not really the creative aspect of mathematics. The creative aspect, the illumination, is not described in, and in fact, lacks formal rigor. As with the realist judge, the conclusion is intuited and the demonstration is later worked out.<sup>51</sup>

45. GEORGE POLYA, PATTERNS OF PLAUSIBLE INFERENCE 158 (1954).

46. *Id.*

47. LET US TEACH GUESSING (George Polya film produced for Comm. on Educ. Media, Mathematical Ass'n of America, by Kurt Simon Productions, 1965).

48. John Littlewood, *The Mathematician's Art of Work*, 5 ROCKEFELLER U. REV. 1, 3 (1967), reprinted in COLLECTED PAPERS OF J.E. LITTLEWOOD 1625, 1627 (1982). Justice Cardozo used similar language in his discussion of how judges reach legal conclusions, writing of the "successive stages of preparation, incubation and illumination." BENJAMIN CARDOZO, THE PARADOXES OF LEGAL SCIENCE 59-60 (1928), reprinted in SELECTED WRITINGS OF BENJAMIN NATHAN CARDOZO 251, 286 (Margaret E. Hall ed., 1947).

49. Littlewood, *supra* note 48, at 3.

50. *Id.*

51. Professor Mark Kac makes much the same point. He distinguishes between strategy and tactics of proof, which he also rephrases as the distinction between motivation and execution. He notes with



Not only do mathematicians not, in fact, normally start with the premises and axioms to see what may be generated; such an approach, even in theory, is implausible. The mathematician Henri Poincaré describes mathematical creation:

It does not consist in making new combinations with mathematical entities already known. Anyone could do that, but the combinations so made would be infinite in number and most of them absolutely without interest. To create consists precisely in not making useless combinations and in making those which are useful and which are only a small minority. Invention is discernment, choice.<sup>52</sup>

He, too, finds the occurrence of sudden illumination in mathematical creation striking, and finds the role of unconscious activity uncontested.<sup>53</sup>

While other mathematicians could be cited,<sup>54</sup> this part of the examination will close with the views of the philosopher Charles Sanders Peirce.

It is difficult to decide between the two definitions of mathematics; the one is by its method, that of drawing necessary conclusions; the other by its aim and subject matter, as the study of hypothetical states of things. The former makes or seems to make the deduction of consequences of hypotheses the sole business of the mathematician as such. But it cannot be denied that immense genius has been exercised in the mere framing of . . . hypotheses.<sup>55</sup>

The non-mathematician may view the work of the mathematician as sitting down with a set of axioms and deriving results from those axioms. While the result must be shown to be derived from the axioms, one must have a result in mind when the deductive effort begins or the likelihood of producing an important piece of mathematics would be similar to the likelihood of the proverbial monkey randomly typing the sonnet. The direction of the deductive effort and the result toward which

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regret the fact that published mathematics stresses the execution and largely ignores the motivation. MARK KAC, ENIGMAS OF CHANCE: AN AUTOBIOGRAPHY 2 (1985).

52. Henri Poincaré, *Mathematical Creation*, 4 THE WORLD OF MATHEMATICS 2041, 2043 (James R. Newman ed., 1956).

53. *Id.* at 2045.

54. See, e.g., PAUL R. HALMOS, I WANT TO BE A MATHEMATICIAN: AN AUTOMATHOGRAPHY 321 (1985) ("Mathematics is not a deductive science . . . . When you try to prove a theorem, you don't just list the hypotheses, and then start to reason. What you do is trial and error, experimentation, guesswork."); James J. Sylvester, *The Study That Knows Nothing of Observation*, in 3 THE WORLD OF MATHEMATICS 1758, 1759 (James R. Newman ed., 1956) ("[M]athematical analysis is constantly invoking the aid of new principles, new ideas, and new methods, not capable of being defined by any form of words, but springing direct from the inherent powers and activity of the human mind, and from continually renewed introspection of that inner world of thought . . . ."); see also LUDWIG WITTGENSTEIN, REMARKS ON THE FOUNDATIONS OF MATHEMATICS proposition I-167, 47e (G.H. von Wright et al. eds., 1956) (trans. by G.E.M. Anscombe) ("The mathematician is an inventor, not a discoverer.").

55. Charles S. Peirce, *The Essence of Mathematics*, 3 THE WORLD OF MATHEMATICS 1773, 1779 (James R. Newman ed., 1956).

the proof is directed must precede the proof itself. The source of the proposed result is not deduction, it is intuition, illumination, or hunch.

The American realists' view of the work of the judge is similar. The realist denies that the judge sits down with a set of legal rules or principles and a set of facts relevant to the case at hand and subsequently derives a result from those rules and facts. Rather, the judge has a result in mind before the opinion writing effort begins. That result is also reached by intuition, illumination, or hunch.

In both mathematics and in the realists' view of law, the already accepted conclusion provides the direction for the construction of a form of deductive demonstration. Both the mathematician and the judge work backward. The mathematician determines what lesser results would suffice to prove the intended result, what is required to prove the lesser results, and so on until reaching a recognized set of axioms or a set of propositions the mathematician is willing to accept as a basis for his or her work.<sup>56</sup> The realist judges claim to have done the same. With result in mind, the judge works backwards to reach accepted principles or rules that justify the result.

Both the mathematician and the judge then mask the true nature of their efforts. When the proof is written, it begins with the statement of axioms and assumptions and proceeds step by step to the result. The creative effort that went into recognizing the result as being both of interest and potentially provable is ignored. When the opinion is written, it begins with a recitation of the facts,<sup>57</sup> sets out rules and principles of law, and demonstrates that the result logically follows from the facts and law. That the result was first reached by hunch or intuition is hidden, and seemingly denied, by the deductive form of the opinion.

It is interesting to note that at least one of the American realist judges noted the similarity between at least the initial intuited stages of law and mathematics. Judge Hutcheson, in discussing intuition or hunch, wrote:

[I]t is that tiptoe faculty of the mind which can feel and follow a hunch which makes not only the best gamblers, . . . lawyers . . . [and] judges . . . , but it is the same faculty which has guided and will continue to guide the great scientists of the world, and even those august dealers in certitude, the mathematicians themselves, to their most difficult solutions . . . .<sup>58</sup>

What remains is to see what these similarities say about law and legal analysis.

56. A mathematician working in group theory, for example, may not be able to derive an intuited result from the axioms of group theory alone. But, it may be that the addition of another assumption would suffice. If the result is important enough and there are a sufficiently interesting number of groups meeting the additional assumption, a result applying to that special area of group theory would be significant.

57. It is, of course, only the relevant facts that are presented, and the choice of facts as relevant would also be argued to be result driven.

58. Hutcheson, *supra* note 26, at 279.

### III. What Does the Comparison Say About Rules of Law?

If the fact that the realist judges confessed to working backwards from conclusion to principles and rules is to lead to the conclusion that there are no rules of law, it would appear that the fact that mathematicians work in the same direction would lead to the conclusion that there are no mathematical rules or principles. No one makes such a claim of mathematics. Indeed, mathematics is the paradigm of a formal system. Whatever the direction of the mathematician's initial work, the result is accepted as mathematics only when the later demonstration is presented and the result is seen to flow from the application of rules of inference to recognized axioms or principles.<sup>59</sup> Thus, unless there is some relevant difference between mathematical and legal reasoning, it would be questionable to conclude that there are no rules of law.

There are, of course, differences between law and mathematics. Mathematics is a formal system with a rigor that would appear not possible in law. Holmes warned of "[t]he danger [in] the notion that a given system, ours, for instance, can be worked out like mathematics from some general axioms of conduct."<sup>60</sup> Even jurists who argue that there are rules for legal analysis that provide for the existence of right answers to difficult legal questions do not assert that those rules have the logical rigor of mathematics.<sup>61</sup>

It might be argued that one relevant difference between law and mathematics is that law is incomplete. There are gaps in the law, and at least a part of the judicial function is said to be legislating at the interstices of the law.<sup>62</sup> These gaps necessarily occur, because legislation cannot be written in sufficient detail to include all possibilities. Law might then be seen as differing from mathematics in that, while one will always (at least in principle) be able to prove or disprove a mathematical proposition, there will always be legal questions the answers to which are not controlled by statute or prior case law.

The flaw in this argument is that it does not, as Godel's Theorem demonstrates,<sup>63</sup> distinguish law from mathematics. Since even as a rule governed system such as

59. There has been some recent concern over the presentation of a demonstration. The recent solution to the four color problem — the question of whether or not four colors are sufficient to color, with no abutting areas the same color, any map meeting certain conditions — involved computer programs that did not allow for a step by step examination of a deductive demonstration. See, e.g., Edward Swart, *The Philosophical Implications of the Four-Color Problem*, 87 AM. MATHEMATICAL MONTHLY 697 (1980); Thomas Tymoczko, *The Four-Color Problem and Its Philosophical Significance*, 76 J. PHIL. 57 (1979).

60. O.W. Holmes, *The Path of the Law*, 10 HARV. L. REV. 457, 465 (1897).

61. Professor Dworkin, for example, allows decisions to be based not only on the explicit holdings of past opinions but from the moral principles those decisions presuppose as justification. See, e.g., RONALD DWORKIN, *LAW'S EMPIRE* 96 (1986). Such mining out of underlying moral principles would seem to lack mathematical rigor, although the search would still be for moral rules adopted by earlier judges.

62. See *Southern P. Co. v. Jensen*, 244 U.S. 205, 220 (1917) (Holmes, J.).

63. See *supra* notes 65-68 and accompanying text.

mathematics<sup>64</sup> is incomplete, the incompleteness of law should not lead to the conclusion that law is not rule governed. It also must be remembered what Godel's Theorem actually demonstrates. The fact that there are holes in any axiomatization of arithmetic does not mean that there are no truths of arithmetic. There are plenty of arithmetic propositions that can be proved true or false. Godel's Theorem only tells us that there are also an infinity of such propositions that can be proven neither true nor false. Most of the important propositions fall into the provable class,<sup>65</sup> because the axioms are chosen to provide a basis for what is known to be important to arithmetic.

Even if Godel's Theorem were to carry over to law,<sup>66</sup> it should prove no more in law than it does in mathematics. While the theorem may have dealt a blow to the Hilbert program of developing a formalization that would capture all of mathematics, no one seriously proposes such a formalization of law. While such an application of the theorem to law would also demonstrate that there will be legal questions that are undecidable within existing law, it is already generally accepted that there are *some* hard cases, and Godel's Theorem would not demonstrate that *all* cases are hard cases. As in arithmetic, the important issues might be resolvable.<sup>67</sup>

Where there are hard cases, a new "axiom," a new rule of law, must be added to deal with the issue. Similarly in mathematics, new axioms must be added if holes are to be filled. Again, the addition of the axiom of arithmetic will fill only some of the holes, and there will remain an infinity of undecidable propositions. The same will be true of the addition of the new rule of law. The existence of holes has not led mathematicians to despair; there are mathematical propositions that are easy to prove. It should not lead jurists to despair; there might still be easy cases.

The way in which new rules of law are selected to fill the holes in the law is, of course, extralegal.<sup>68</sup> However, the same must be true of arithmetic. The choice of an additional axiom would have to be motivated by the conclusions one wishes to be able to derive. That choice cannot be determined with the arithmetic system, because the proposition at issue was undecidable within that system. Once again the fact that the rules and axioms of arithmetic don't cover everything, and that new rules must sometimes be invented, does not lead to the conclusion that there are no rules in mathematics and should not lead to the parallel conclusion in law.

64. The characterization of mathematics as rule governed is not to deny the role of intuition nor to suggest, contrary to Godel's Theorem, that any proposition or its negation is provable from a set of axioms. Rather, the characterization asserts only that there are well defined rules of inference from accepted sets of axioms.

65. *But see infra* notes 73-76 and accompanying text.

66. *See supra* notes 4-10 and accompanying text.

67. This is not to deny the existence of other reasons why all cases might be hard cases. Godel's Theorem does not speak to the critics' claim that conflicting psychological commitments lead to irresolvable conflict in law, *see supra* note 42 and accompanying text, or to the claim that language is so inherently ambiguous that rules cannot be given definitive readings, *see supra* notes 43-44 and accompanying text.

68. Even Professor Dworkin agrees that the judge's political morality plays a role in the decision of hard cases. *See, e.g.,* DWORKIN, *supra* note 61, at 255-56.

Perhaps the major difference between the two fields is in whether and how one can know he or she is incorrect in the initial intuition, illumination, or hunch. Mathematicians recognize the regularity with which the intuition is in error. Poincaré finds such instances to be fairly regular.

I have spoken of the feeling of absolute certitude accompanying the inspiration; in the cases cited this feeling was no deceiver, nor is it usually. But do not think this a rule without exception; often this feeling deceives us without being any less vivid, and we only find it out when we seek to put on foot the demonstration.<sup>69</sup>

Professor Halmos seems to find the experience of the incorrect intuition even more regular.

After tossing and turning a while . . . I 'solve' the problem; the proof or the counterexample arrives in a flash of insight and, content, I roll over and fall asleep. Almost always the flash turns out to be spurious; the proof has a gigantic hole in it or the counterexample is not counter anything. . . . On a few occasions it even turned out to be right.<sup>70</sup>

The mathematician cannot always, however, tell when the intuition is incorrect. Certainly, if in attempting to prove a proposition one manages instead to prove its negation or finds a counterexample to the proposition,<sup>71</sup> then it is clear that the intuition was incorrect. According to Poincaré, in such instances of flawed illumination "we almost always notice that this false idea, had it been true, would have gratified our natural feeling for mathematical elegance."<sup>72</sup>

There are important mathematical propositions that have been under examination for years, or even centuries, but have never been proved. Probably, the two most notable examples, and certainly the most simply stated, are Goldbach's conjecture and Fermat's "theorem." Goldbach's conjecture<sup>73</sup> is that every even number is the sum of two prime numbers. Fermat's "theorem"<sup>74</sup> is that, where  $n$  is greater than 2, there are no whole numbers,  $a$ ,  $b$  and  $c$  for which  $a^n + b^n = c^n$ . Despite a great deal of effort on the part of generations of mathematicians, no proof exists for either

69. Poincaré, *supra* note 52, at 2046.

70. HALMOS, *supra* note 54, at 324. Lest Professor Halmos be thought a lesser mathematician, it should be noted that he was writing about fooling himself into going to sleep when his subconscious was working on a problem. The intuitions he generated in that attempt may not have had the intensity of those of Poincaré and he may have accepted them to put his mind to rest.

71. A counterexample is a set of facts for which a proposition is false. As a simple example, the number two serves as a counterexample to the proposition *all prime numbers are odd*.

72. Henri Poincaré, *supra* note 52, at 2048.

73. See Edward Kasner & James R. Newman, *Pastimes of Past and Present Times*, in 4 THE WORLD OF MATHEMATICS 2416, 2437-38 (James R. Newman ed., 1956).

74. See Eric T. Bell, *The Queen of Mathematics*, in 1 THE WORLD OF MATHEMATICS 498, 509 (James R. Newman ed., 1956).

proposition.<sup>75</sup> Neither, however, has a counterexample for either been presented, and proofs limited to numbers of certain sizes have been presented.<sup>76</sup>

The difference here between law and mathematics seems to be that it may be less clear that the legal hunch or intuition is incorrect. While there are some mathematical hunches or intuitions that have never been shown correct or incorrect, at least it is not uncommon that a mathematical intuition is shown to be incorrect. On the other hand, it may be argued that the legal intuition can never be shown to be incorrect.

As Professor Kennedy and other scholars in the critical legal studies tradition have argued, law may be deeply conflicted.<sup>77</sup> If in every case there are conflicting principles that may be brought to bear, then perhaps one's hunch can never be demonstrated to be incorrect. Any intuition at which one arrives could be derived from such a set of conflicted or inconsistent legal principles.<sup>78</sup> Of course, the opposite intuition can also be so derived. From whatever hunch one begins, one can always work one's way back to a set of premises to justify that hunch. Any conclusion can be demonstrated, and hence, in a sense, no conclusion can be established.

While the crits could be correct in their view of the existence of inner conflict and its effect on law, that conclusion does not flow from the observation that it may be more difficult to recognize error in legal hunches than in mathematical intuitions. As seen, any errors in the hunches of Fermat or Goldbach are just as difficult to detect or demonstrate.

Further, Hayek would respond to this difficulty in reconciling conflict in law, not by agreeing that there are no rules of law, but instead by arguing that we simply have difficulty expressing the law as it exists.<sup>79</sup> In his view of law as a spontaneous social order, law has evolved with human societies and indeed has been a factor in which groups survive, but humans have never been able to provide complete statements of the law.<sup>80</sup> What would appear to be inconsistencies are simply inability to explain the rules that resolve conflicts between competing principles. For Hayek, these rules exist but have never been completely captured in legal opinion or statute. While not wanting to adopt this view, it does offer an explanation for conflict other than the nonexistence of legal rules.

75. Fermat claimed, in a marginal note in one of his books, that he had a proof for his "theorem." Fermat's proof, if it was in fact a valid proof, has not survived. *See id.* Princeton University mathematician Andrew Wiles has recently offered a proof of Fermat's Theorem. While promising, the proof is still undergoing examination for validity. *See* John Schwartz, *This Equation Figures to Answer a 17th-Century Puzzle*, WASHINGTON POST, Aug. 2, 1993, at A3.

76. *See* Bell, *supra* note 74, at 510. Despite the fact that these propositions are unproven, they may be used so long as any results derived are recognized as based on the assumption that the proposition is true.

77. *See* Duncan Kennedy, *Form and Substance in Private Law Adjudication*, 89 HARV. L. REV. 1685 (1976). *See generally* KELMAN, *supra* note 37, at 15-63.

78. It is also true of a formal system, such as mathematics, that if a set of assumptions contains two contradictory propositions, then any conclusion may be derived from the set of assumptions.

79. *See* FRIEDRICH A. VON HAYEK, *LAW, LEGISLATION AND LIBERTY* 72-93 (1973) (chapter 4).

80. *See id.*

One last comparison of the mathematician with the realist judge draws on the role of intuition in mathematical discovery. That role was mentioned by both the mathematicians<sup>81</sup> and the judges<sup>82</sup> examined above. An interesting look at the mathematics role is provided by Professor Douglas Hofstadter's examination of idiot savants and Srinivasa Ramanujan, one of the century's greatest mathematicians.<sup>83</sup> The idiot savant, more kindly the autistic savant, may perform very complex calculations — for example, multiplying two 100-digit numbers mentally, without being able to explain the process employed. Hofstadter states that psychological research concludes that such individuals do not receive flashes of enlightenment or occult or divine inspiration, though it may seem that way to the individual savant. Rather, they race through the sequential steps of the calculation with more speed and self-confidence than the rest of us.<sup>84</sup>

Ramanujan had similar ability, but at a much more advanced and abstract level. He is said to have been able simply to state important and complex results. Sometimes he could give some insight into the process<sup>85</sup> but sometimes claimed inspiration from the goddess Namagiri in his dreams.<sup>86</sup> Ramanujan had a familiarity with mathematical entities that allowed him seemingly to intuit difficult results, but perhaps the better characterization of his mental activities is that he was able to race through the sorts of generalizations and analogies employed by more ordinary mathematicians, and by non-mathematicians, with such speed and at such a subconscious level, that he could not explain the reasoning process.

Such an explanation would fit Professor Littlewood's explanation of illumination as "the emergence of the creative idea into the conscious . . . imply[ing] some mysterious rapport between the subconscious and the conscious."<sup>87</sup> It also meshes with Professor Sylvester's description of mathematics "springing direct from the inherent powers and activity of the human mind, and from continually renewed introspection of that inner world of thought . . ."<sup>88</sup>

If the process by which the mathematician intuits the conclusion, before working backward to the premises, is a subconscious process guided by experience with those premises, the same may well be true of even the realist judge. If a judge has a great deal of experience ruminating about the law, that ruminating would seem more likely to affect his or her legal conclusions than the rumination involving the judge's breakfast. If the hunch or intuition is guided by experience in the law, then the hunch or experience is, if not rule governed, at least rule affected.

Consequently, the judge and the mathematician both seem to work backwards from the hunch, but both may in a sense be working back to their actual starting

81. See *supra* notes 45, 50, 53 and accompanying text.

82. See *supra* notes 27-33 and accompanying text.

83. DOUGLAS R. HOFSTADTER, GODEL, ESCHER, BACH: AN ETERNAL GOLDEN BRAID 562-67 (1979).

84. *Id.* at 567.

85. See *id.* at 565 (discussing a solution involving complex fractions).

86. *Id.* at 566.

87. Littlewood, *supra* note 48, at 3.

88. See *supra* note 54.

points. When the subconscious route from those starting points to the hunch followed an acceptable path, from the standpoint of mathematical or legal reasoning, that reasoning may be reversed to find the starting points. The judge or mathematician must then reconstruct, as much as can be done, the earlier subconscious route to the conclusion. The fact that the reconstruction may be backward does not necessarily imply that the subconscious road to the original "hunch" was not forward from the premises or at least experience with the premises.

The mathematician may find that he or she was mistaken. The subconscious route may have jumped a chasm to reach a conclusion to which no acceptable mathematical road may be constructed. Perhaps the difference between the good and the poor mathematician is the regularity with which their subconscious musings leap these breaks. The mathematician who spends a career trying to build unconstructable logical paths will not be recognized as a great mathematician. The mathematician who recognizes a few important, valid results and constructs the roads to those results will be so recognized.

For the judge, the chasms are not as obvious. The judge is less likely to be confronted by the critic demonstrating the logical impossibility of a step in the judge's reconstruction of the subconscious route. Nonetheless, some opinions are clearly more strained than others. Perhaps the good judge, like the good mathematician, is the judge whose subconscious follows routes to conclusions that really do follow from the legal rules. When that judge reconstructs the route in an opinion, the opinion reads well. When the poorer judge follows a subconscious route to a poor conclusion, a route back might still be constructed. While that route will not reach an obvious chasm as is found in the erroneous mathematical hunch, it may lead to a strained opinion that, while not logically wrong, may be recognized as legally wrong.

#### *IV. Conclusion*

The point of this short article is not to completely refute the outgrowths of legal realism. The scholars of the critical legal studies movement offer interesting and important psychological arguments that indicate that conflict in the law mirrors psychological conflict within the individual. To whatever degree the individual conflict is irresolvable, so too would seem to be the conflict of legal principles. Similarly, those who argue from the indeterminacy of language also offer interesting arguments over the variety of meanings that might be assigned to any statement of a legal rule.

What has been attacked is the import of the initial observations that started the realists and the later critics and indeterminists down the road to the claims that there are no legal rules. The realists' observation of appearing to work backwards from judgment to opinion is similar to the observations of mathematicians that they too work backwards. Since mathematics would seem to be the paradigm of a rule governed system, that simple observation of the perceived direction of effort cannot be taken as proof that there are no rules, either in mathematics or in law.



